- investigate and model the behaviour of standing waves on strings and/or in pipes to relate quantitatively the fundamental and harmonic frequencies of the waves that are produced to the physical characteristics (e.g. length, mass, tension, wave velocity) of the medium

Standing Waves and Harmonics

- Harmonic frequencies, or harmonics, are specific frequencies of vibration that produce standing waves which results in specific sounds which sound pleasant.

– What are standing waves?

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- The lowest resonant frequency of a vibrating object is called its fundamental frequency.

– Define resonant frequency.

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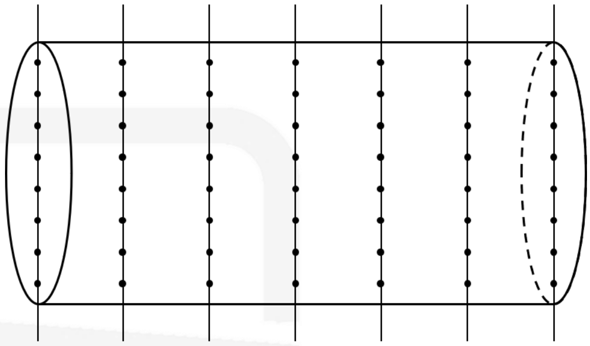
- Most vibrating objects have more than one resonant frequency and those used in musical instruments typically vibrate at harmonics of the fundamental as part of a harmonic series.
  - A harmonic is defined as an integer multiple of the fundamental frequency.
  - After the first fundamental frequency, harmonics are known as *overtones*.
- At any frequency other than a harmonic frequency, standing waves are not established and instead, the resulting disturbance is irregular and non-repeating.
- The sounds produced from musical instruments are due to standing waves.

STANDING WAVES IN OPEN TUBES

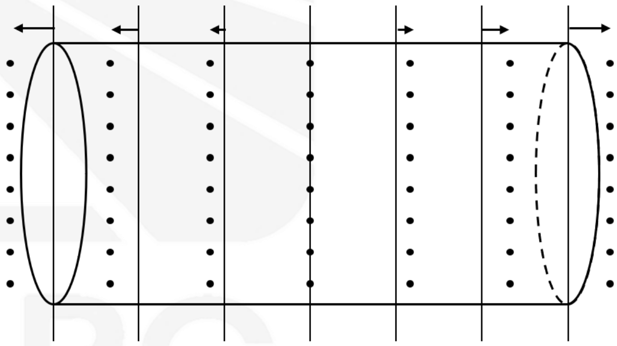
- In an open tube, when a sound wave passes through which causes the disturbance of air particles inside, a standing wave is set-up.
- Below shows the vibrational motion of the air particles to form a standing wave.

The particles are undisturbed and positioned at the equilibrium, marked by the vertical lines.

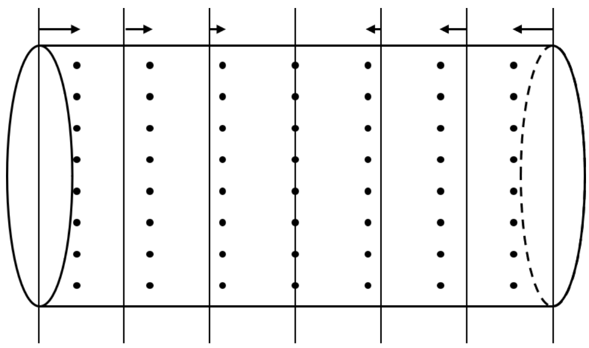


The particles are at their respective amplitudes. Observe that the column of particles in the middle do not displace (NODE).

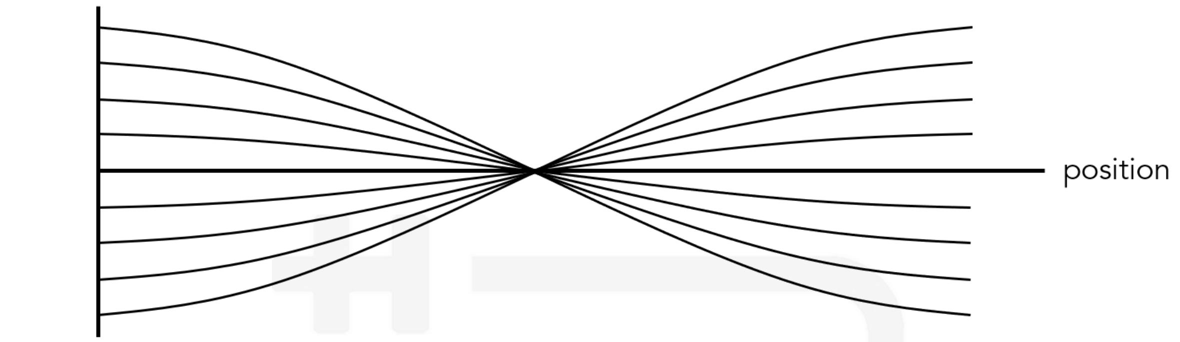
In contrast, the particles at either end have the greatest amplitude (ANTI-NODES).



The particles oscillate to the other side of the equilibrium, thus showing their oscillatory motion.



- By graphing the oscillatory motion of the air particles in the open tube, the standing wave becomes more apparent. Let displacement to the left of the equilibrium position be considered negative.



Interpret the graph above to depict the standing wave inside the open tube.

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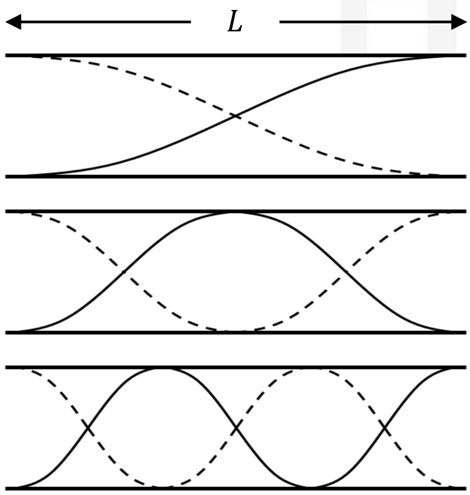
- Thus, standing waves formed in an open tube are characterised by anti-nodes at either end (graphically represent as a CREST).



$\frac{1}{2}\lambda_1 = L, \qquad \lambda_1 = 2L$

$\lambda_2 = L$

$\frac{3}{2}\lambda_3 = L, \qquad \lambda_3 = \frac{2}{3}L$

$$\lambda_n = \frac{2L}{n}$$

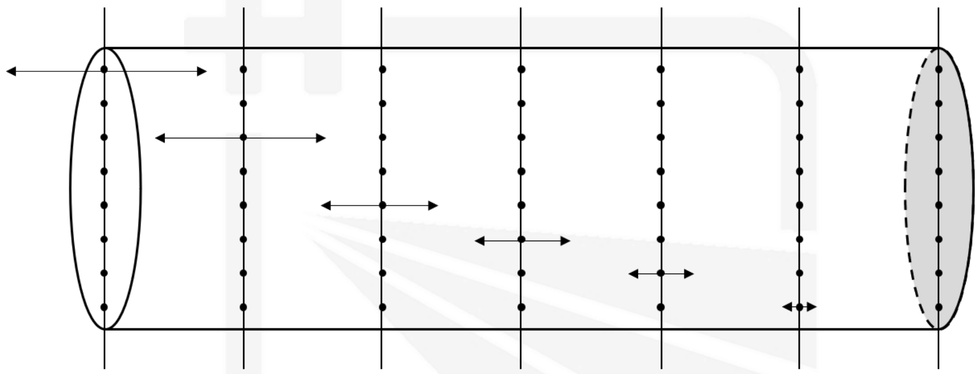
$$n = 1, 2, 3, \ldots$$

STANDING WAVES IN CLOSED TUBES

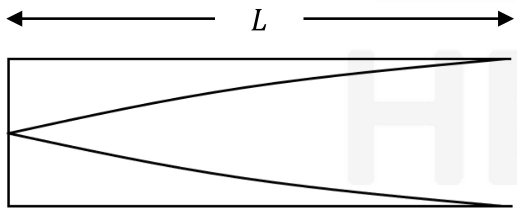
- Standing waves can also form in a tube where one side is closed off.

- Particles at the open end will be able to vibrate with maximum amplitude (just as we have learnt previously).

- Particles at the closed end, however, will not be able to displace from the equilibrium position at all since they will bump into the surface and transfer (lose) all its kinetic energy.

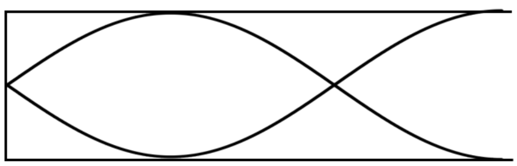


- As a result, the particles at the open end will always be the ANTI-NODE while the particles at the closed end will always be the NODE of the standing wave.



$\frac{1}{4}\lambda_1 = L, \qquad \lambda_1 = 4L$

First harmonic
(fundamental)

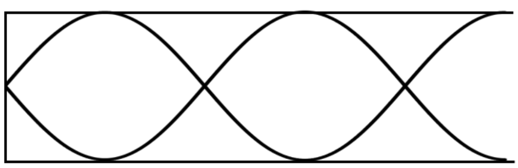


$\frac{3}{4}\lambda_2 = L, \qquad \lambda_2 = \frac{4}{3}L$

Third harmonic
(first overtone)

$\lambda_n = \frac{4L}{n}$

$n = 1, \cancel{2}, 3, \cancel{4}, 5 \ldots$

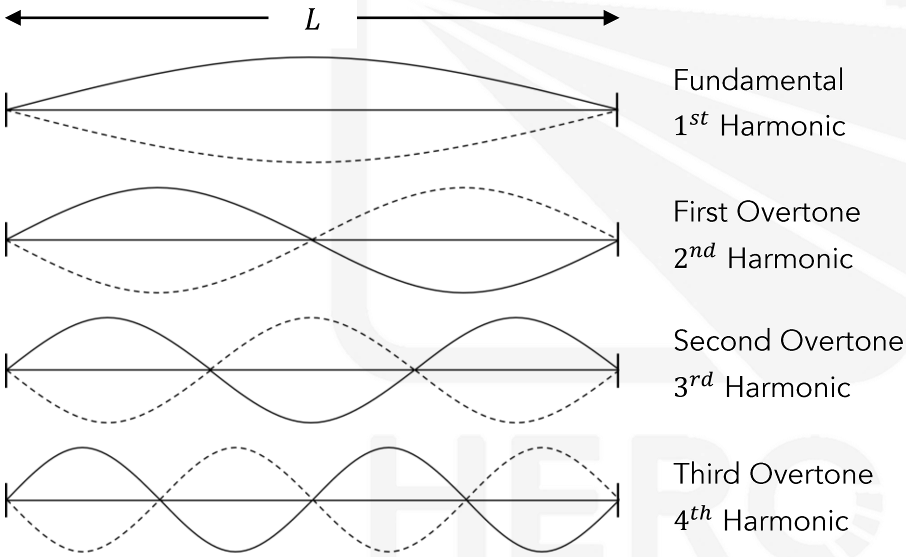


$\frac{5}{4}\lambda_2 = L, \qquad \lambda_2 = \frac{4}{5}L$

Fifth harmonic
(third overtone)

STANDING WAVES IN FIXED ENDS

- Strings in musical instruments have fixed ends (confined space with two boundaries).
- When the string is plucked, the vibrations will travel down the length of the string and reflect at the boundary repeatedly, resulting in a multitude of wave cycles travelling in both directions.
- Such waves will interfere with each other. At particular frequencies, their superposition will induce an interference pattern that appears to be stationary (i.e. transverse standing wave).
- As the particles at the two fixed ends are not able to displace from equilibrium, they are both NODES.



$$\lambda_n = \frac{2L}{n}$$

$$n = 1, 2, 3 \ldots$$

– Given that the length of the string, L, is $10\ m$, calculate the wavelength of the standing wave formed in each harmonic.

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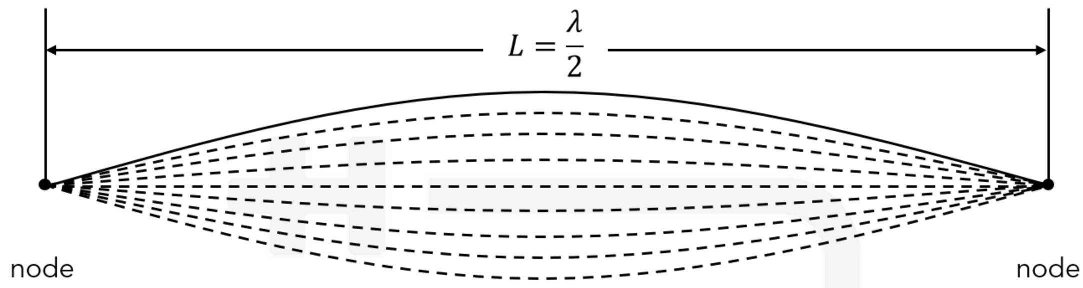
TENSION

- The fundamental vibrational mode of a stretched string with two fixed ends (nodes) is such that the wavelength is twice the length of the strings (as was covered in the previous page).



- There are two important formulae that you are required to know, one of which has already been covered:

  i. Basic wave relationship:

$$f_1 = \frac{v}{2L}$$

  - f_1 = frequency of 1^{st} harmonic (fundamental mode) (Hz)
  - v = velocity of the wave in the string (ms^{-1})
  - L = length of the string (m)

  ii. Wave velocity:

$$v = \sqrt{\frac{T}{m/L}}$$

  - v = velocity of the wave in the string (ms^{-1})
  - T = tension on the string (N)
  - m/L = mass per unit length (kg/m)

- By rearranging the wave relationship to subject v and substituting it into the velocity equation, we will derive the FREQEUNCY EXPRESSION.

$$f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}$$

- From the above equation, we can see that $f_1 \propto \sqrt{T}$. Hence, the greater the tension in the string, the greater the frequency of vibration of the fundamental frequency (i.e. 1^{st} harmonic).

Explain why increasing tension, T, will increase the frequency vibration of the fundamental mode of a string that is stretched and fixed at both ends.

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Why do we only derive the frequency expression for the fundamental mode when a string will vibrate in a range of overtones that form a harmonic series ($n = 1, 2, 3$ ...)?

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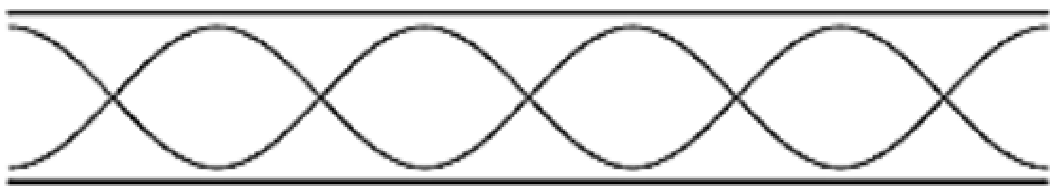
Concept Check 2.6 (7 marks)

The speed of sound in air is $3.40 \times 10^2\ ms^{-1}$.

Carlie plays the recorder. The recorder can be modelled as an open pipe. On one occasion, the note Carlie plays has the following standing wave pattern for one of its overtones (harmonics).



The length of the pipe when she plays this note is $28.5\ cm$.

(a) Calculate the wavelength of the standing wave shown in the diagram. **2**

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(b) Which harmonic (or overtone) is shown in the diagram above? **1**

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(c) By first calculating the wavelength the fundamental standing wave would have in this length of pipe (or otherwise), calculate the frequency of the fundamental standing wave. **2**

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(d) Explain how the fundamental standing wave is produced in this pipe. **2**

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